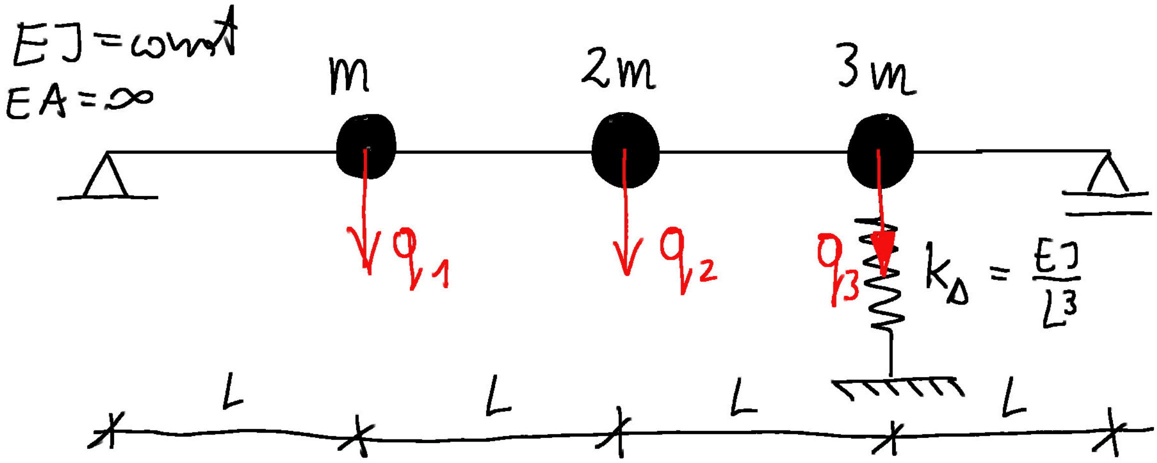


$n_h = 1$

$n_g = 3 + 3 = 6$ $\qquad$ $n_h = 1 < 3 = n_{gd}$

$d = 3$ $\qquad$ $\Downarrow$

$n_{gd} = 3$ $\qquad$ $\underline{\underline{MS}}$

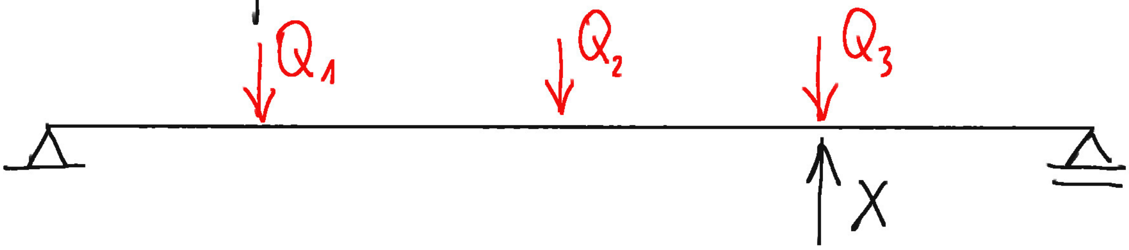
1. Energia kinetyczna układu – wyznaczenie macierzy $\mathbb{B}$

$$E_k = \frac{1}{2} m \dot{q}_1^2 + \frac{1}{2} 2m \dot{q}_2^2 + \frac{1}{2} 3m \dot{q}_3^2$$

$$E_k = \frac{1}{2}\left(b_{11}\dot{q}_1^2 + b_{12}\dot{q}_1\dot{q}_2 + b_{13}\dot{q}_1\dot{q}_3 + b_{21}\dot{q}_2\dot{q}_1 + b_{22}\dot{q}_2^2 + \right.$$
$$\left. + b_{23}\dot{q}_2\dot{q}_3 + b_{31}\dot{q}_3\dot{q}_1 + b_{32}\dot{q}_3\dot{q}_2 + b_{33}\dot{q}_3^2 \right)$$

$$\mathbb{B} = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 3m \end{bmatrix}$$

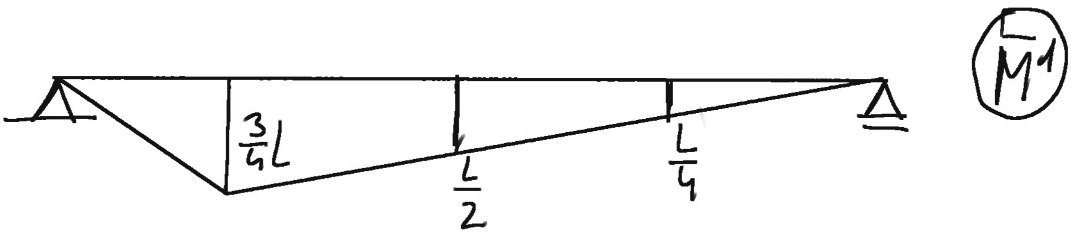
2. Wyznaczenie macierzy podatności w bazie porzenowej $\check{\mathbb{D}}$

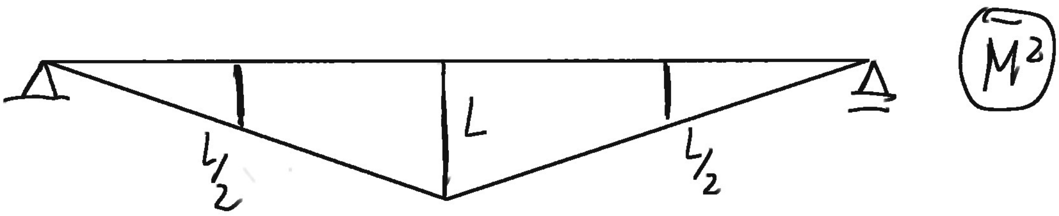


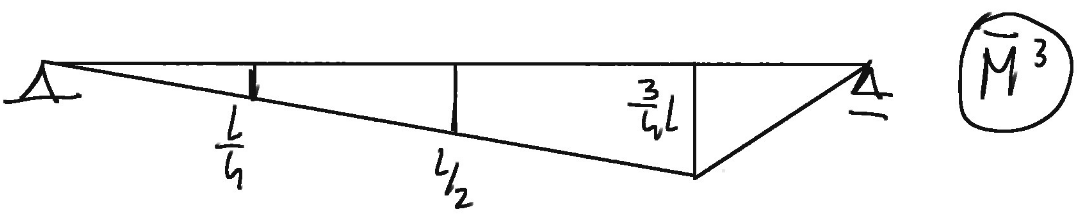
$$\begin{bmatrix} \mathbb{D}_{QQ} & \mathbb{D}_{QX} \\ \mathbb{D}_{XQ} & \mathbb{D}_{XX} \end{bmatrix} \begin{bmatrix} \bar{Q} \\ X \end{bmatrix} = \begin{bmatrix} \mathbb{D}\bar{Q} \\ -\frac{1}{k_\Delta} \cdot X \end{bmatrix}$$

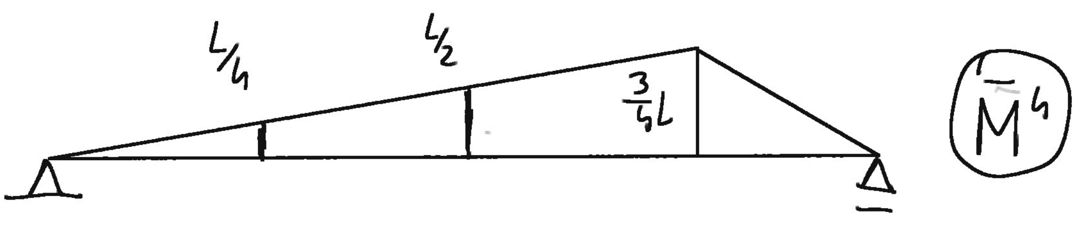
$$\begin{bmatrix} \mathbb{D}_{QQ} & \mathbb{D}_{QX} \\ \mathbb{D}_{XQ} & \left(\mathbb{D}_{XX} + \frac{1}{k_\Delta}\right) \end{bmatrix} \begin{bmatrix} \bar{Q} \\ X \end{bmatrix} = \begin{bmatrix} \boxed{\mathbb{D}}\bar{Q} \\ 0 \end{bmatrix}$$

$$\mathbb{D} = \mathbb{D}_{QQ} - \mathbb{D}_{QX} \cdot \left(\mathbb{D}_{XX} + \frac{1}{k_\Delta}\right)^{-1} \cdot \mathbb{D}_{XQ}$$









$$\delta_{11} = \frac{1}{EJ}\frac{1}{2}\cdot L\cdot\frac{3}{4}L\cdot\frac{2}{3}\cdot\frac{3}{4}L + \frac{1}{EJ}\frac{1}{2}\cdot 3L\cdot\frac{3}{4}L\cdot\frac{2}{3}\cdot\frac{3}{4}L = \frac{3L^3}{4EJ}$$

$$\delta_{12} = \delta_{21} = \frac{1}{EJ}\frac{1}{2}\cdot L\cdot\frac{3}{4}L\cdot\frac{2}{3}\cdot\frac{L}{2} + \frac{L}{6EJ}\left(\frac{3}{4}L\cdot\frac{L}{2} + 4\cdot\frac{5}{8}L\cdot\frac{3}{4}L + \frac{L}{2}\cdot L\right) +$$

$$+ \frac{L}{6EJ}\left(\frac{L}{2}\cdot L + 4\cdot\frac{3}{8}L\cdot\frac{3}{4}L + \frac{L}{4}\cdot\frac{L}{2}\right) + \frac{1}{EJ}\frac{1}{2}\cdot L\cdot\frac{L}{4}\cdot\frac{2}{3}\cdot\frac{L}{2} =$$

$$= \frac{41L^3}{48EJ} = 0{,}916(6)\frac{L^3}{EJ}$$

$$\delta_{13} = \delta_{31} = 2\frac{1}{EJ}\frac{1}{2}\cdot L\cdot\frac{3}{4}L\cdot\frac{2}{3}\cdot\frac{L}{4} + 2\frac{L}{6EJ}\left(\frac{3}{4}L\cdot\frac{L}{2} + 4\cdot\frac{5}{8}L\cdot\frac{3}{8}L + \frac{L}{2}\cdot\frac{L}{2}\right) +$$

$$= 0{,}6458(3)\frac{L^3}{EJ}$$

$$\delta_{14} = \delta_{41} = -0{,}6458(3)\frac{L^3}{EJ}$$

$$\delta_{22} = \frac{1}{EJ}\frac{1}{2}\cdot 2L\cdot L\cdot\frac{2}{3}\cdot L\cdot 2 = 1{,}333(3)\frac{L^3}{EJ}$$

$$\delta_{23} = \delta_{32} = 0{,}916(6)\frac{L^3}{EJ}$$

$$\delta_{24} = \delta_{42} = -0{,}916(6)\frac{L^3}{EJ}$$

$$\delta_{33} = \frac{3}{4}\frac{L^3}{EJ}$$

$$\delta_{34} = \delta_{43} = -\frac{3L^3}{4EJ}$$

$$\delta_{44} = \frac{3L^3}{4EJ}$$

$$\check{\mathbb{D}} = \frac{L^3}{EJ}\begin{bmatrix} 0{,}75 & 0{,}916(6) & 0{,}583(3) & -0{,}583(3) \\ & 1{,}333(3) & 0{,}916(6) & -0{,}916(6) \\ & & 0{,}75 & -0{,}75 \\ \text{sym.} & & & 0{,}75 \end{bmatrix}$$

$$\mathbb{D} = \mathbb{D}_{QQ} - \mathbb{D}_{QX} \cdot \mathbb{D}_{XX}^{-1} \cdot \mathbb{D}_{XQ}$$

$$\mathbb{D} = \frac{L^3}{EJ}\begin{pmatrix} 0.555578 & 0.611084 & 0.333314 \\ 0.611084 & 0.853211 & 0.523771 \\ 0.333314 & 0.523771 & 0.428571 \end{pmatrix}$$

$$\det(\mathbb{D}\mathbb{B} - \lambda I) = 0$$

$$\frac{L^3}{EJ}\mathbb{D}^* = \mathbb{D} \quad , \quad \mathbb{B} = m \cdot \mathbb{B}^*$$

$$\mathbb{B}^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\boxed{\omega = \sqrt{\frac{EJ}{\lambda m L^3}}}$$

$$\omega_1 = 0{,}559\sqrt{\frac{EJ}{mL^3}}$$

$$\omega_2 = 1{,}866\sqrt{\frac{EJ}{mL^3}}$$

$$\omega_3 = 4{,}064\sqrt{\frac{EJ}{mL^3}}$$

$$\bar{W}_1 \sim \operatorname{adj}(D^* \cdot B^* - \lambda_1 I)$$

$$\operatorname{adj}\left(\begin{pmatrix} 0.555578 & 0.611084 & 0.333314 \\ 0.611084 & 0.853211 & 0.523771 \\ 0.333314 & 0.523771 & 0.428571 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} - 3{,}2 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$\operatorname{adj}\begin{pmatrix} -2.64442 & 1.22217 & 0.999943 \\ 0.611084 & -1.49358 & 1.57131 \\ 0.333314 & 1.04754 & -1.91429 \end{pmatrix} =$$

$$= \begin{bmatrix} 1{,}213 & +1{,}694 & 1{,}138 \\ +3{,}387 & 4{,}729 & +3{,}178 \\ 3{,}414 & +4{,}766 & 3{,}203 \end{bmatrix}^T =$$

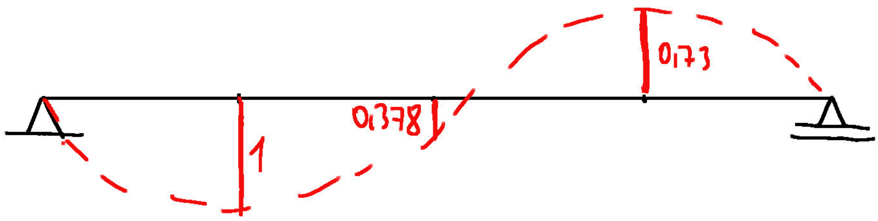
$$= \begin{bmatrix} 1{,}213 & 3{,}387 & 3{,}414 \\ 1{,}694 & 4{,}729 & 4{,}766 \\ 1{,}138 & 3{,}178 & 3{,}203 \end{bmatrix}$$

$$\bar{W}_1 = \begin{bmatrix} 0{,}716 \\ 1 \\ 0{,}672 \end{bmatrix}$$

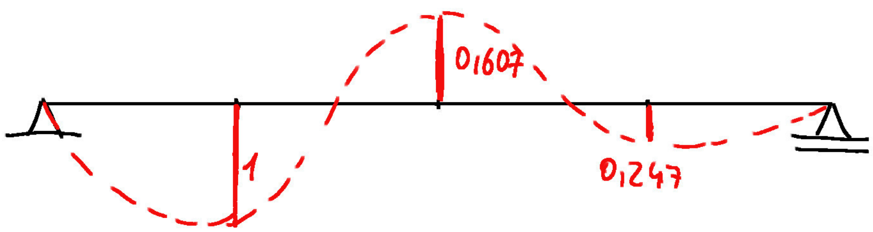
$$\bar{W}_2 = \begin{bmatrix} 1 \\ 0{,}378 \\ -0{,}73 \end{bmatrix}, \quad \bar{W}_3 = \begin{bmatrix} 1 \\ -0{,}607 \\ 0{,}247 \end{bmatrix}$$

$$W = \begin{bmatrix} 0{,}716 & 1 & 1 \\ 1 & 0{,}378 & -0{,}607 \\ 0{,}672 & -0{,}73 & 0{,}247 \end{bmatrix}$$

forma 1



forma 2



forma 3