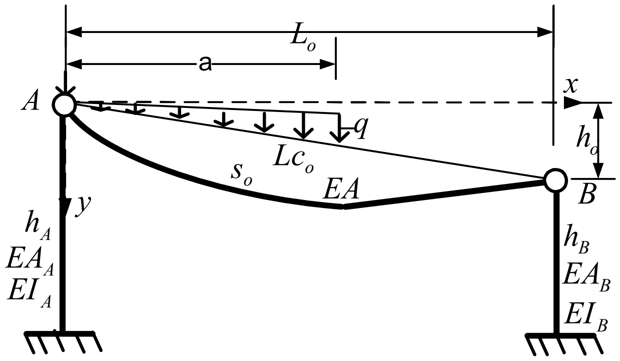
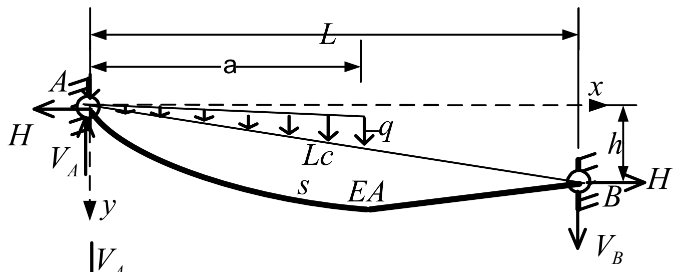
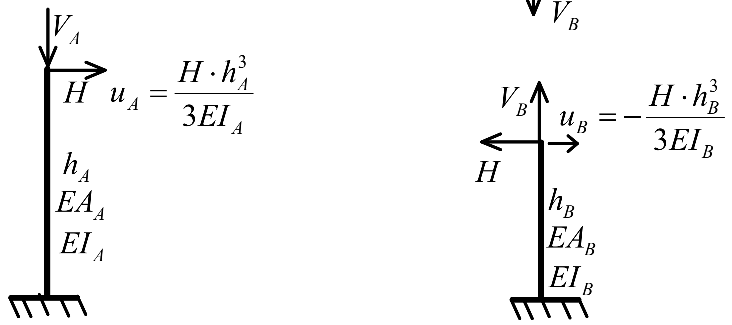
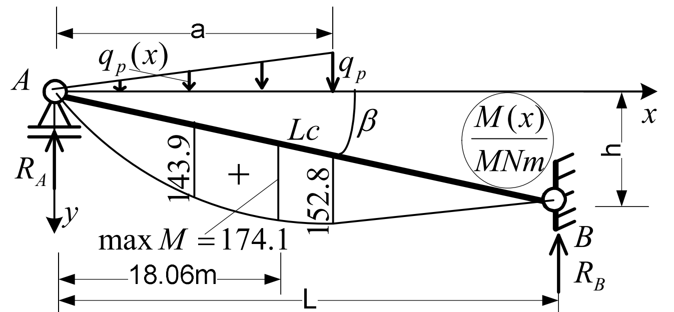
Dla cięgna jak na rys. obok wyznaczyć długość cięgna s, kształt jego zwisu $f(x)$ i ekstremalne siły osiowe: $\max N$, $\min N$ z uwzględnieniem zmiany temperatury o $\Delta T = -80^o C$ oraz podatności giętnej słupów i wydłużenia cięgna Δs w wyniku działającego obciążenia mając dane: $EA = 2000MN$
$EI_A = 300000 MN^2 m^2$, $EI_B = 80000 MN^2 m^2$,
$L_o = 40m$, $h_o = 8m$, $h_A = 16m$, $h_B = 8m$ $s_o = 46m$,
$a = 23m$, $q = 2MN/m$, $\alpha_T = 0.0001/^o C$.

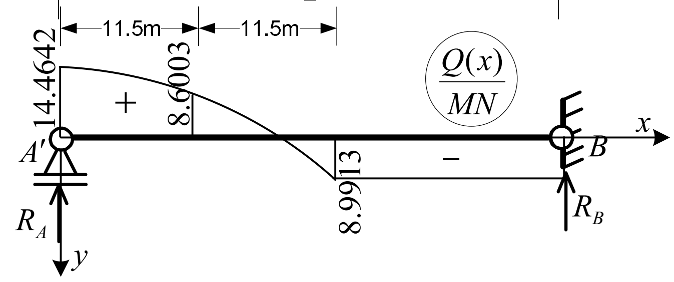


1.BELKI ZASTĘPCZE I OBLICZENIA WSTĘPNE









Parametry geometryczne

$Lc_o = \sqrt{L_o^{\ 2} + h_o^{\ 2}} = 40.792m,$

$\cos\beta_o = L_o / Lc_o = 0.9806,$

$\cos^2\beta_o = 0.9615, \ \cos^3\beta_o = 0.9429.$

Rozwiązanie belek zastępczych

- Obciążenie sprowadzone do poziomej linii rozłożenia

$$\frac{q \cdot a / \cos\beta}{2} = \frac{q_p \cdot a}{2} \Rightarrow$$

$$q_p = \frac{q}{\cos\beta} = 2.04\text{MN/m}, \quad \frac{q_p}{a} = \frac{q_p(x)}{x}$$

$$\Rightarrow q_p(x) = q_p \cdot \frac{x}{a} = 0.0887 MN/m^2 \cdot x,$$

- Reakcje, momenty zginające i siły tnące

$$\sum M_B = R_A \cdot L - \frac{q_p \cdot a}{2} \cdot \left(L - \frac{2}{3}a\right) = 0 \quad \Rightarrow$$

$$R_A = \frac{q_p \cdot a}{2} \cdot \left(1 - \frac{2 \cdot a}{3 \cdot L}\right) =$$

$$= \frac{2.04 MN/m \cdot 23m}{2} \cdot \left(1 - \frac{2 \cdot 23m}{3 \cdot 40m}\right) = 14.4642 MN,$$

$$\sum M_A = R_B \cdot L_o - \frac{q_p \cdot a}{2} \cdot \frac{2}{3} a = 0 \quad \Rightarrow$$

$$R_B = \frac{q_p \cdot a^2}{3 \cdot L} = \frac{2.04 MN/m \cdot 23^2 m^2}{3 \cdot 40m} =$$

$8.9913 MN$,

$$\sum Y = q_p \cdot a/2 - R_A - R_B = 2.04 MN/m \cdot 23m/2 - 14.4642 - 8.9913 = 0,$$

$$M(x) = R_A \cdot x - \frac{q_p(x) \cdot x}{2} \cdot \frac{x}{3} = R_A \cdot x - \frac{q_p \cdot x^3}{6a} = \left(R_A - \frac{q_p \cdot x^2}{6a}\right) \cdot x,$$

$$M(a/2) = \left(14.4642 MN - 2.04 MN/m \cdot 23^2 m^2 / 4 / (6 \cdot 23m)\right) \cdot 23m/2 = 143.8560 MNm,$$

$$M(a) = \left(14.4642 MN - 2.04 MN/m \cdot 23^2 m^2 / (6 \cdot 23m)\right) \cdot 23m = 152.8169 MNm,$$

$Q(x) = R_A - q_p(x)\cdot x/2 = R_A - \frac{q_p \cdot x^2}{2\cdot a}/(2\cdot a),$

$Q(x)=0$ dla $x(Q=0) = \sqrt{\frac{2\cdot a\cdot R_A}{q_p}} = \sqrt{\frac{2\cdot 23m\cdot 14.4642MN}{2.04MN/m}} = 18.0615m,$

$\max M(x) = M(18.0615m) = \left(14.4642 - \frac{2.04MN/m\cdot 18.0615^2 m^2}{6\cdot 23m}\right)MN\cdot 18.0615m = 174.1465MNm,$

$Q_A = Q(0) = R_A = 14.4642MN,$

$Q(a/2) = 14.4642MN - 2.04MN/m\cdot 23^2 m^2/2/(2\cdot 23m) = 8.6003MN,$

$Q(a) = 14.4642MN - 2.04MN/m\cdot 23^2 m^2/(2\cdot 23m) = -8.9913MN,$ $Q_B = -R_B = -8.9913MN.$

$\max Q(x) = Q_A = 14.4642MN,$ $\min Q(x) = Q(a) = Q_B = -8.9913MN,$

Obliczenie całki z Q^2

$(Q(x))^2 = R_A^2 - \frac{R_A\cdot q_p}{a}\cdot x^2 + \frac{q_p^2}{4a^2}\cdot x^4,$

$\int_0^a (Q(x))^2\cdot dx = \left(R_A^2\cdot x - \frac{R_A\cdot q_p}{3a}\cdot x^3 + \frac{q_p^2}{20a^2}\cdot x^5\right)\Big|_0^a = \left(R_A^2 - \frac{R_A\cdot q_p\cdot a}{3} + \frac{q_p^2\cdot a^2}{20}\right)\cdot a = 2140.581MN^2m,$

$\int_a^L (Q(x))^2\cdot dx = (L-a)\cdot R_B^2 = 1374.330MN^2m,$

$\int_0^a (Q(x))^2\cdot dx + \int_a^L (Q(x))^2\cdot dx = (2140.581+1374.330)MN^2m = 3514.911MN^2m.$

2.ITERACYJNE WYZNACZENIE H, s, u_A I u_B

Wartości wyjściowe do iteracji: $s_o := s_o\cdot(1+\alpha_T\cdot\Delta T) = 46\cdot(1+0.0001\cdot(-80))m = 45.632m$,

$L_o = 40m$, $h_o = 8m$, $\cos\beta_o = 0.9806$, $\cos^2\beta_o = 0.9615$, $\cos^3\beta_o = 0.9429$.

Wzory: $H_i = \sqrt{\frac{0.5\cdot\cos^3\beta_{i-1}\cdot\int_0^L (Q(x))^2\,dx}{\left(s_{i-1} - \frac{L_{i-1}}{\cos\beta_{i-1}}\right)}} \Rightarrow \Delta s_i = \frac{1}{EA}\left(\frac{H_i\cdot L_{i-1}}{\cos^2\beta_{i-1}} + \frac{1}{H_{i-1}}\int_0^L (Q(x))^2\cdot dx\right),\ s_i = s_o + \Delta s_i$

$u_{Ai} = \frac{H_i\cdot h_A^3}{3EI_A},\quad u_{Bi} = \frac{H_i\cdot h_B^3}{3EI_B} \Rightarrow L_i = L_o - u_{Ai} + u_{Bi},\quad h_i = h_o - v_{Ai} + v_{Bi},\quad Lc_i = \sqrt{L_i^2 + h_i^2} \Rightarrow$

$\cos\beta_i = \frac{L_i}{Lc_i},\quad \cos^2\beta_i,\quad \cos^3\beta_i.$

1 przyb. $H_1 = \sqrt{\frac{0.5\cdot\cos^3\beta_o\cdot\int_0^L (Q(x))^2\,dx}{\left(s_o - \frac{L_o}{\cos\beta_o}\right)}} = \sqrt{\frac{0.5\cdot 0.9429\cdot 3514.91}{(45.632 - 40/0.9806)}}MN = 18.50340MN,$

$\Delta s_1 = \frac{1}{EA}\left(\frac{H_1\cdot L_o}{\cos^2\beta_o} + \frac{1}{H_1}\int_0^L (Q(x))^2\cdot dx\right) = \left(\frac{18.5034\cdot 40}{0.9615\cdot 2000} + \frac{3514.91}{2000\cdot 18.5034}\right)m = 0.47985m,$

$s_1 = s_o + \Delta s_i = 45.632m + 0.47985m = 46.11185m,$

$u_{A_1} = \frac{H_1\cdot h_A^3}{3EI_A} = \frac{18.50340MN\cdot 16^3 m^3}{3\cdot 300000MNm^2} = 0.08421m,\quad u_{B_1} = -\frac{H_1\cdot h_B^3}{3EI_B} = -\frac{18.50340MN\cdot 8^3 m^3}{3\cdot 80000MNm^2} = -0.03947m,$

$$L_1 = L_o - u_{A_1} + u_{B_1} = 40m - 0.08421 - 0.03947m = 39.87632m, \qquad h_1 = h_o = 8m,$$

$$Lc_1 = \sqrt{L_1^2 + h_1^2} = \sqrt{39.87632^2 + 8^2}\,m = 40.67088m,$$

$$\cos\beta_1 = \frac{L_1}{Lc_1} = 0.98046, \quad \cos^2\beta_1 = 0.96131, \quad \cos^3\beta_1 = 0.94253.$$

2 przyb. $$H_2 = \sqrt{\frac{0.5\cdot\cos^3\beta_1\cdot\int_0^L (Q(x))^2\,dx}{\left(s_1 - \dfrac{L_1}{\cos\beta_1}\right)}} = \sqrt{\frac{0.5\cdot 0.9425\cdot 3514.91}{(46.11185 - 39.8763/0.9805)}}MN = 17.44551MN,$$

$$\Delta s_2 = \frac{1}{EA}\left(\frac{H_2\cdot L_1}{\cos^2\beta_1} + \frac{1}{H_2}\int_0^L (Q(x))^2\cdot dx\right) = \left(\frac{17.44551\cdot 39.8763}{0.9613\cdot 2000} + \frac{3514.91}{2000\cdot 17.44551}\right)m = 0.46257m,$$

$$s_2 = s_o + \Delta s_2 = 45.632m + 0.46257m = 46.09457m,$$

$$u_{A2} = \frac{H_2\cdot h_A^3}{3EI_A} = \frac{17.44551\cdot 16^3}{3\cdot 300000} = 0.07940m, \qquad u_{B2} = -\frac{H_2\cdot h_B^3}{3EI_B} = -\frac{17.44551MN\cdot 8^3}{3\cdot 80000} = -0.03722m,$$

$$L_2 = L_o - u_{A2} + u_{B2} = 40m - 0.07940 - 0.03722m = 39.88338m,$$

$$Lc_2 = \sqrt{L_2^2 + h_2^2} = \sqrt{39.88338^2 + 8^2}\,m = 40.67781m,$$

$$\cos\beta_2 = \frac{L_2}{Lc_2} = 0.98047, \quad \cos^2\beta = 0.96132,\ \cos^3\beta = 0.94255.$$

3 przyb. $$H_3 = \sqrt{\frac{0.5\cdot\cos^3\beta_2\cdot\int_0^L (Q(x))^2\,dx}{\left(s_2 - \dfrac{L_2}{\cos\beta_2}\right)}} = \sqrt{\frac{0.5\cdot 0.9425\cdot 3514.91}{(46.09457 - 39.8838/0.98047)}}MN = 17.4869MN,$$

$$\Delta s_3 = \frac{1}{EA}\left(\frac{H_3\cdot L_2}{\cos^2\beta_2} + \frac{1}{H_3}\int_0^L (Q(x))^2\cdot dx\right) = \left(\frac{17.4869\cdot 39.8763}{0.9613\cdot 2000} + \frac{3514.91}{2000\cdot 17.48691}\right)m = 0.46325m,$$

$$s_3 = s_o + \Delta s_3 = 45.632m + 0.46325m = 46.09525m,$$

$$u_{A3} = \frac{H_3\cdot h_A^3}{3EI_A} = \frac{17.4869\cdot 16^3}{3\cdot 300000} = 0.07958m, \qquad u_{B3} = -\frac{H_3\cdot h_B^3}{3EI_B} = -\frac{17.4869MN\cdot 8^3}{3\cdot 80000} = -0.03731m,$$

$$L_3 = L_o - u_{A3} + u_{B3} = 40m - 0.07958 - 0.03731m = 39.88311m,$$

$$Lc_3 = \sqrt{L_3^2 + h_3^2} = \sqrt{39.88311^2 + 8^2}\,m = 40.67753m,$$

$$\cos\beta_3 = \frac{L_3}{Lc_3} = 0.9805, \ \cos^2\beta_3 = 0.9613, \ \cos^3\beta_3 = 0.94255.$$

4 przyb. $$H_4 = \sqrt{\frac{0.5\cdot\cos^3\beta_3\cdot\int_0^L (Q(x))^2\,dx}{\left(s_3 - \dfrac{L_3}{\cos\beta_3}\right)}} = \sqrt{\frac{0.5\cdot 0.9425\cdot 3514.91}{(46.09525 - 39.88311/0.9805)}}MN = 17.5854MN,$$

$$\Delta s_5 = \frac{1}{EA}\left(\frac{H_5\cdot L_4}{\cos^2\beta_4} + \frac{1}{H_5}\int_0^L (Q(x))^2\cdot dx\right) = \left(\frac{17.5854\cdot 39.88311}{0.9613\cdot 2000} + \frac{3514.91}{2000\cdot 17.5854}\right)m = 0.46322m,$$

$$\Delta s_4 = \frac{1}{EA}\left(\frac{H_4\cdot L_3}{\cos^2\beta_3} + \frac{1}{H_4}\int_0^L (Q(x))^2\cdot dx\right) = \left(\frac{17.5854\cdot 39.88311}{0.9613\cdot 2000} + \frac{3514.91}{2000\cdot 17.5854}\right)m = 0.46322m,$$

$$s_4 = s_o + \Delta s_4 = 45.632m + 0.46322m = 46.09522m,$$

$$u_{A4}=\frac{H_4\cdot h_A^3}{3EI_A}=\frac{17.5854\cdot 16^3}{3\cdot 300000}=0.07958m,\quad u_{B4}=-\frac{H_4\cdot h_B^3}{3EI_B}=-\frac{17.5854MN\cdot 8^3}{3\cdot 80000}=-0.03730m,$$

Ponieważ $u_{A4}=u_{A3}$, i $u_{B4}=u_{B3}$, to i $L_4=L_3=39.88311m$,

$$Lc_4=Lc_3=40.6775m,$$

$$\cos\beta_4=\cos\beta_3=0.9805,\quad \cos^2\beta_4=0.9613,\ \cos^3\beta_4=0.94255.$$

5 przyb. $$H_5=\sqrt{\frac{0.5\cdot\cos^3\beta_4\cdot\int_0^L (Q(x))^2\,dx}{\left(s_4-\frac{L_4}{\cos\beta_4}\right)}}=\sqrt{\frac{0.5\cdot 0.9425\cdot 3514.91}{(46.09522-39.88312/0.9805)}}MN=17.5854MN,$$

$$\Delta s_5=\frac{1}{EA}\left(\frac{H_5\cdot L_4}{\cos^2\beta_4}+\frac{1}{H_5}\int_0^L (Q(x))^2\cdot dx\right)=\left(\frac{17.5854\cdot 39.88311}{0.9613\cdot 2000}+\frac{3514.91}{2000\cdot 17.5854}\right)m=0.46322m,$$

Z faktu, że $H_5=H_4$ i $\Delta s_5=\Delta s_4$ wynika, że pozostałe wartości w tym przyblizeniu będą równe wynikom przybliżenia poprzedniego, co oznacza, że wyniki przybliżenia 4-tego są wynikami końcowymi rozwiązania zadania: $H=17.4854MN$, $\Delta s=0.4632m$, $s=46.0952m$, $u_A=0.0796m$, $u_B=-0.0373m$, $\cos\beta=0.9805$.

3.EKSTREMALNE WARTOŚCI SIŁ I ZWIS CIĘGNA

$$\max V(x)=\max Q(x)+H\cdot tg\beta=(14.4642+17.4854\cdot 0.2)MN=17.9613MN,$$

$$\min V(x)=\min Q(x)+H\cdot tg\beta=(-8.9913+17.4854\cdot 0.2)MN=-5.4942MN,$$

Jak widać, V(x) zmienia znak, więc $\min(\mathrm{V(x)})^2=0$

$$V(x)=0 \quad \text{dla} \quad Q(x)=-H\cdot tg\beta \quad \Rightarrow \quad x(V=0)=\sqrt{\frac{2a\cdot(R_A+H\cdot tg\beta)}{q_p}}=$$

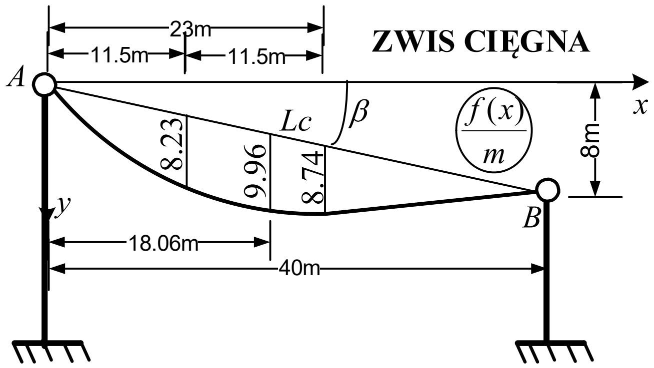
$$=\sqrt{\frac{2\cdot 23m\cdot(14.4642MN+17.4854\cdot 0.2)}{2.04MN/m}}=20.1248m\ ,$$

$$\max N=\sqrt{H^2+\max(V(x))^2}=\sqrt{17.4854^2+17.9613^2}MN=25.0669MN\le N_{dop},$$

$$\min N=\sqrt{H^2+\min(\mathrm{V(x)})^2}=\sqrt{17.4854^2+0^2}MN=17.4854MN>0,$$

$$f(x)=\frac{M(x)}{H}=\frac{M(x)}{17.4854MN},\qquad f(a/2)=\frac{143.8560MNm}{17.4854MN}=8.227m,$$

$$\max f(x)=f(18.0615m)=\frac{174.1465MNm}{17.4854MN}=9.959m,\quad f(a)=\frac{152.8169MNm}{17.4854MN}=8.739m.$$



Maksymalne momenty w słupach: $\max M_{\text{sluplewy}}=H\cdot h_A=17.4854MN\cdot 16m=279.77MNm,$

$$\max M_{\text{slupprawy}}=H\cdot h_B=17.4854MN\cdot 8m=139.89MNm.$$