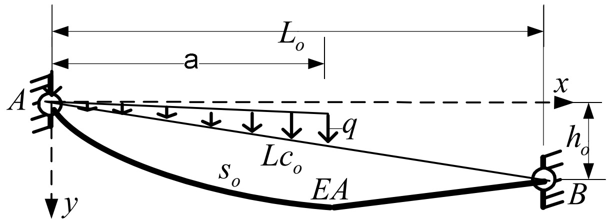
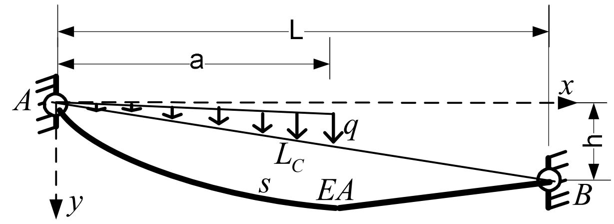
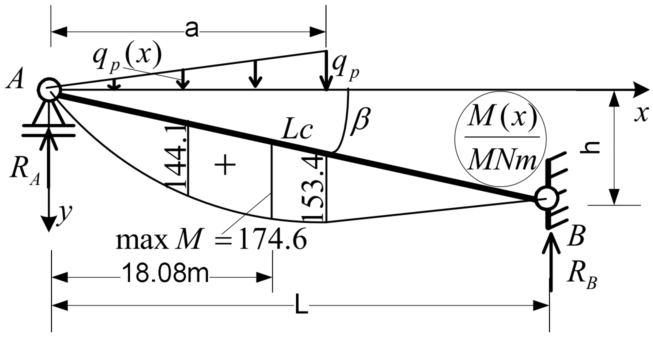
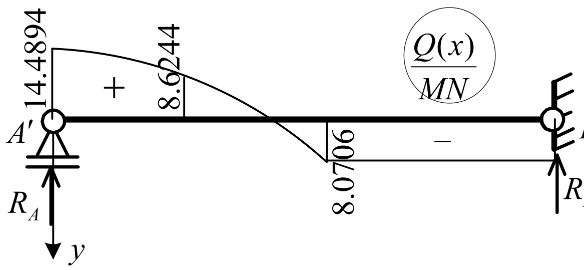
Dla cięgna jak na rys. obok wyznaczyć długość cięgna s, kształt jego zwisu $f(x)$ i ekstremalne siły osiowe: $\max N, \min N$ z uwzględnieniem zmiany temperatury o $\Delta T = -80^o C$ i przemieszczeń podpór $u_A = -10cm$, $v_A = 0$, $u_B = 0$, $v_B = 6cm$ oraz wydłużenia cięgna Δs w wyniku działającego obciążenia mając dane: $EA = 1000MN$, $L_o = 40m$, $h_o = 8m$, $s_o = 46m$, $a = 23m$, $q = 2MN/m$, $\alpha_T = 0.0001/^o C$.



1.BELKI ZASTĘPCZE I OBLICZENIA WSTĘPNE







Korekta geometrii układu uwzględniająca przemieszczenia podpór:

$L = L_o - u_A + u_B = (40-(-0.1)+0)m = 40.1m,$

$h = h_o - v_A + v_B = (8-0+0.06)m = 8.06m,$

$L_C = \sqrt{L^2 + h^2} = 40.902m, \; \cos\beta = L/L_C = 0.9804,$

$\cos^2\beta = 0.9612,$

$\cos^3\beta = 0.9423, \qquad tg\beta = h/L = 0.2010,$

Rozwiązanie belek zastępczych

- Obciążenie sprowadzone do poziomej linii rozłożenia $\dfrac{q\cdot a/\cos\beta}{2} = \dfrac{q_p \cdot a}{2} \Rightarrow$

$q_p = \dfrac{q}{\cos\beta} = 2.04\text{MN/m}, \; \dfrac{q_p}{a} = \dfrac{q_p(x)}{x} \Rightarrow$

$$q_p(x) = q_p \cdot \frac{x}{a} = 0.0887MN/m^2 \cdot x.$$

- Reakcje, momenty zginające i siły tnące

$$\sum M_B = R_A \cdot L - \frac{q_p \cdot a}{2}\cdot\left(L - \frac{2}{3}a\right) = 0 \quad \Rightarrow$$

$$R_A = \frac{q_p \cdot a}{2}\cdot\left(1 - \frac{2\cdot a}{3\cdot L}\right) =$$

$$= \frac{2.04MN/m\cdot 23m}{2}\cdot\left(1 - \frac{2\cdot 23m}{3\cdot 40.1m}\right) = 14.4894MN,$$

$$\sum M_A = R_B \cdot L - \frac{q_p\cdot a}{2}\cdot\frac{2}{3}a = 0 \quad \Rightarrow \quad R_B = \frac{q_p\cdot a^2}{3\cdot L} = \frac{2.04MN/m\cdot 23^2 m^2}{3\cdot 40.1m} = 8.9706MN,$$

$$\sum Y = \frac{q_p\cdot a}{2} - R_A - R_B = \frac{2.04MN/m\cdot 23m}{2} - 14.4894 - 8.9706 = 0$$

$$M(x) = R_A\cdot x - \frac{q_p(x)\cdot x}{2}\cdot\frac{x}{3} = R_A\cdot x - \frac{q_p\cdot x^3}{6a} = \left(R_A - \frac{q_p\cdot x^2}{6a}\right)\cdot x = M(x) = \left(R_A - \frac{q_p\cdot x^2}{6a}\right)\cdot x$$

$$= \left(14.4894 - \frac{2.04\cdot x^2}{6\cdot 23m^2}\right)MN\cdot x = \left(14.4894 - \frac{x^2}{67.6471m^2}\right)MN\cdot x,$$

$$M(a/2) = \left(14.4894 - \frac{23^2/4}{67.6471}\right)MN\cdot\frac{23}{2}m = 144.1456MNm,$$

$$M(a) = \left(14.4894 - \frac{23^2}{67.6471}\right)MN\cdot 23m = 153.3965MNm,$$

$$Q(x) = R_A - \frac{q_p(x)\cdot x}{2} = R_A - \frac{q_p \cdot x^2}{2\cdot a}, \qquad x = \sqrt{\frac{2\cdot a\cdot R_A}{q_p}}$$

$$Q(x) = 0 \quad \text{dla } x(Q=0) = \sqrt{\frac{2\cdot a\cdot R_A}{q_p}} = \sqrt{\frac{2\cdot 23m\cdot 14.4894MN}{2.04MN/m}} = 18.0754m$$

$$\max M(x) = M(18.0754m) = \left(14.4894 - \frac{18.0754^2}{67.6471}\right)MN\cdot 18.0754m = 174.6016MNm,$$

$$Q_A = Q(0) = R_A = 14.4894MN, \qquad Q(a/2) = R_A - \frac{q_p\cdot a}{8} = 8.6244MN,$$

$$Q(a) = R_A - \frac{q_p\cdot a}{2} = -8.9706MN, \qquad Q_B = -R_B = -8.9706MN.$$

$$\max Q(x) = Q_A = 14.4894MN, \qquad \min Q(x) = Q(a) = Q_B = -8.9706MN,$$

Obliczenie całki z Q^2

$$\left(Q(x)\right)^2 = R_A^2 - \frac{R_A\cdot q_p}{a}\cdot x^2 + \frac{q_p^2}{4a^2}\cdot x^4,$$

$$\int_0^a \left(Q(x)\right)^2\cdot dx = \left(R_A^2\cdot x - \frac{R_A\cdot q_p}{3a}\cdot x^3 + \frac{q_p^2}{20a^2}\cdot x^5\right)\Bigg|_0^a = \left(R_A^2 - \frac{R_A\cdot q_p\cdot a}{3} + \frac{q_p^2\cdot a^2}{20}\right)\cdot a = 2148.273MN^2m,$$

$$\int_a^L \left(Q(x)\right)^2\cdot dx = \left(L-a\right)\cdot R_B^2 = 1376.057MN^2m,$$

$$\int_0^a \left(Q(x)\right)^2\cdot dx + \int_a^L \left(Q(x)\right)^2\cdot dx = (2148.273+1376.057)MN^2m = 3524.33MN^2m.$$

2.ITERACYJNE WYZNACZENIE H i s

Wartość wyjściowadługości cięgna z uwzględnieniem zmiany temperatury:

$$s_o := s_o\cdot(1+\alpha_T\cdot\Delta T) = 46\cdot(1+0.0001\cdot(-80))m = 45.632m$$

Wzory:

$$H_i = \sqrt{\frac{0.5\cdot\cos^3\beta\cdot\int_0^L \left(Q(x)\right)^2 dx}{\left(s_{i-1} - \frac{L}{\cos\beta}\right)}} = \frac{40.7491}{\sqrt{s_{i-1}/m - 40.902}}MN,$$

$$\Delta s_i = \frac{1}{EA}\left(\frac{H_i\cdot L}{\cos^2\beta} + \frac{1}{H_i}\int_0^L \left(Q(x)\right)^2\cdot dx\right) = \left(\frac{H_i\cdot 40.1}{0.9612\cdot 1000MN} + \frac{3524.33MN}{1000\cdot H_i}\right)m = \left(\frac{H_i}{23.97MN} + \frac{3.5243MN}{H_i}\right)m$$

$$s_i = s_o + \Delta s_i = 45.632m + \Delta s_i.$$

1 przyb.

$$H_1 = \frac{40.7491}{\sqrt{s_o/m - 40.902}}MN = 18.73667MN,$$

$$\Delta s_1 = \left(\frac{H_1}{23.97MN} + \frac{3.5243MN}{H_1}\right)m = 0.4849m, \qquad s_1 = 45.632m + \Delta s_1 = 46.1169m.$$

2 przyb.

$$H_2 = \frac{40.7491}{\sqrt{s_1/m - 40.902}}MN = 17.84433MN,$$

$$\Delta s_2 = \left(\frac{H_2}{23.97MN} + \frac{3.5243MN}{H_2} \right) m = 0.47099m, \qquad s_2 = 45.632m + \Delta s_2 = 46.10299m.$$

3 przyb.

$$H_3 = \frac{40.7491}{\sqrt{s_2 / m - 40.902}} MN = 17.68818MN,$$

$$\Delta s_3 = \left(\frac{H_3}{23.97MN} + \frac{3.5243MN}{H_3} \right) m = 0.47135m, \qquad s_3 = 45.632m + \Delta s_3 = 46.10335m.$$

4 przyb.

$$H_4 = \frac{40.7491}{\sqrt{s_3 / m - 40.902}} MN = 17.86755MN,$$

$$\Delta s_4 = \left(\frac{H_4}{23.97MN} + \frac{3.5243MN}{H_4} \right) m = 0.47134m, \qquad s_4 = 45.632m + \Delta s_4 = 46.10334m.$$

5 przyb.

$$H_5 = \frac{40.7491}{\sqrt{s_4 / m - 40.902}} MN = 17.86756MN,$$

$$\Delta s_5 = \left(\frac{H_5}{23.97MN} + \frac{3.5243MN}{H_5} \right) m = 0.47134m, \qquad s_5 = 45.632m + \Delta s_5 = 46.1033m.$$

Jak widać wyniki w piątej iteracji praktycznie powtórzyły wyniki z czwartej iteracji, co oznacza koniec iteracji. Jako wyniki końcowe przyjęto: $H = 17.8676MN$, $\Delta s = 0.4713m$, $s = 46.1033m.$

3.EKSTREMALNE WARTOŚCI SIŁ I ZWIS CIĘGNA

$$\max V = \max Q(x) + H \cdot tg\beta = (14.4894 + 17.8676 \cdot 0.2010)MN = 18.0808MN,$$

$$\min V = \min Q(x) + H \cdot tg\beta = (-8.9706 + 17.8676 \cdot 0.2010)MN = -5.3792MN,$$

$$V(x) = 0 \quad \text{dla} \quad x(V = 0) = \sqrt{\frac{2a \cdot (R_A + H \cdot tg\beta)}{q_p}} =$$

$$= \sqrt{\frac{2 \cdot 23m \cdot (14.4894\,MN + 17.8676 \cdot t0.2010)}{2.04\,MN/m}} = 20.192\,m.$$

$$\max N = \sqrt{H^2 + \max(V(x))^2} = \sqrt{17.8676^2 + 18.0808^2}\,MN = 25.4198MN \le N_{dop} = R \cdot A \cdot \gamma,$$

$$\min N = \sqrt{H^2 + (\min absV)^2} = \sqrt{17.8676^2 + 0^2}\,MN = 17.8676MN,$$

$$V(x) = 0 \quad \text{dla} \quad Q(x) = -H \cdot tg\beta,$$

$$f(x) = \frac{M(x)}{H} = \frac{M(x)}{17.8676MN}, \qquad f(a/2) = \frac{144.1456MNm}{17.8676MN} = 8.0675m,$$

$$\max f(x) = f(18.0754m) = \frac{174.6016MNm}{17.8676MN} = 9.7720m,$$

$$f(a) = \frac{153.3965MNm}{17.8676MN} = 8.5852m,$$

ZWIS CIĘGNA